Written Exam at the Department of Economics winter 2016-17
Advanced Industrial Organization
Final Exam
22 December 2016
(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam consists of four (4) pages in total

NB: If you fall ill during the actual examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Please answer all five questions

1. *Durable good monopoly.* A profit-maximizing monopolist sells an indivisible good which lasts for two periods, $t = 1, 2$. There is no depreciation of the good between periods 1 and 2. The good is produced at zero cost. A consumer wants at most one unit of the good; his valuation of the good is denoted θ, and is his private information. Suppose θ is uniformly distributed on $[0, 1]$. The price and the quantity sold in period $t \in \{1, 2\}$ are denoted p_t and q_t. The discount factor is $\delta = 1/2$ (for both the firm and the consumers). Thus, if a consumer of type θ buys the good in period 1, his payoff will be $\theta - p_1 + \frac{1}{2}\theta$. If he instead buys in period 2 his payoff will be $\frac{1}{2}(\theta - p_2)$. If he never buys, his payoff is 0.

(a) Suppose before period 1, the monopolist announces both p_1 and p_2 and this is a firm commitment. How much will he sell in each period? That is, compute q_1 and q_2.

(b) Now suppose the monopolist cannot make any commitments: he must choose p_1 in period 1 and p_2 in period 2. How much will he sell in each period? That is, compute q_1 and q_2. Show that he sells more in period 1 with commitment (part a) than without commitment (part b), and explain intuitively why this is the case.

2. *Non-linear pricing.* A monopolist is selling an indivisible good. The unit production cost is 0. There are two types of consumer, high type and low type, denoted H and L. Each consumer would like at most two units of the good. Their valuations of the good are as follows. The H type would be willing to pay \$40 to get one unit, and \$80 to get two units. The L type would be willing to pay \$35 to get one unit, and \$50 to get two units. (Of course, getting no unit is worth 0). Half of all consumers are H types, the other half are L types. Each consumer's type $\theta \in \{H, L\}$ is his private information. Let $T(q)$ denote the non-linear tariff. That is, a consumer who buys $q \in \{0, 1, 2\}$ units must pay $T(q)$. Obviously, $T(0) = 0$. The monopolist chooses $T(q)$ to maximize his profit.

(a) Find the optimal $T(1)$ and $T(2)$. *Hint:* consider the incentive-compatibility and participation ("individual rationality") constraints.

(b) Does the monopolist offer a quantity discount, i.e., is it true that $T(2)/2 < T(1)/1$?

3. *A two-stage entry-deterrence game.* In stage 1, the incumbent (firm 1) decides whether or not to invest in a new technology. The new technology would cost K, which is an exogenously given amount.[1]

In stage 2 there is Cournot competition: firm 1 and firm 2 simultaneously choose quantities $q_1 \geq 0$ and $q_2 \geq 0$. The price is $p = 9 - (q_1 + q_2)$. Firm 2's unit production cost is 6, so his profit will be $\pi_2 = (p - 6)q_2$. Firm 1's unit

[1] Notice that the investment decision is binary. You can think of the investment as buying a patent that allows the incumbent to use a new cost-reducing technology. The patent costs K. He either buys the patent or he doesn't buy the patent.

production cost depends on whether he has the new technology. If he made the investment in stage 1, his unit production cost in stage 2 will be 1; otherwise his unit production cost will be 6. Firm 1's profit will therefore be $\pi_1 = (p-6)q_1$ if he did not invest, and $\pi_1 = (p-1)q_1 - K$ if he did invest. We say that *entry is deterred* if firm 2 chooses $q_2 = 0$.

(a) Find the largest K such that entry is deterred in subgame-perfect equilibrium. That is, find K^* such that entry is deterred if and only if K is less than K^*.

(b) Suppose K is slightly smaller than K^* (found in part a) so that entry is deterred. Is the incumbent using what Fudenberg-Tirole would call a "Top Dog" strategy? Explain. *Hint*: A good answer would take into account the following: Are actions in stage 2 strategic substitutes or complements? Does the investment make firm 1 tough or soft? Is there over-investment in equilibrium?

4. *Green-Porter model of collusion.*

(a) Give a brief intuitive explanation for why price wars happen in the Green-Porter model.

(b) Recall that in Porter (1983), the econometric model used for estimating firm conduct is

$$\begin{aligned}
\text{(demand)}\;\; \log(Q_t) &= \alpha_0 + \alpha_1 \log(P_t) + \alpha_2 L_t + u_{1t} \\
\text{(supply)}\;\; \log(P_t) &= \beta_0 + \beta_1 \log(Q_t) + \beta_2 S_t + \beta_3 I_t + u_{2t}
\end{aligned} \tag{1}$$

where P_t and Q_t are price and quantity of railroad transportation, L_t is the dummy for Lake open, I_t is the regime shifter ($I_t = 0$ for competition and $I_t = 1$ for collusion), and S_t is the dummy vector of firm entry/exit to the railroad cartel. We are interested in the "conduct" parameter θ, which is implied by β_3:

$$\beta_3 = -\log(1 + \frac{\theta}{\alpha_1}). \tag{2}$$

Suppose that we have obtained all parameter estimates, $\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$. Show how to use the formula (2) to recover the conduct parameter θ. Then explain how this conduct parameter can be sensitive to the estimate of price elasticity. Specifically, in which direction would a biased α_1 result in a biased estimate of θ?

5. (a) *Standard logit.* The random utility of alternative j given by the standard logit model is

$$u_{ij} = \beta' X_j - \alpha p_j + \epsilon_{ij}$$

where X_j is the vector of observable characteristics, p_j is the price, and ϵ_j accounts for all unobservables (to the econometrician). The choice set is $\mathcal{J} = \{1, ..., J\}$ (no outside option). The idiosyncratic shock ϵ_{ij} follows an i.i.d. extreme value type I distribution. Suppose k is a substitutable good, $k \in \mathcal{J}$. For

$j \in \mathcal{J}$, the conditional choice probability is

$$\sigma_j = \frac{\exp\left(\beta' X_j - \alpha p_j\right)}{\sum_{q \in \mathcal{J}} \exp\left(\beta' X_q - \alpha p_q\right)}$$

and the cross-price elasticity is $e_{jk} = \dfrac{\partial \sigma_j}{\partial p_k} \dfrac{p_k}{\sigma_j}$. Show that IIA (Independence of Irrelevant Alternatives) holds for the standard logit model, and explain why this implies unrealistic substitution patterns.

(b) *Mixed logit.* Now suppose that individuals have different price sensitivities. The random utility is given as

$$u_{ij} = \beta' X_j - \alpha_i p_j + \epsilon_{ij}$$

There are two types of consumers, rich (R) and poor (P). Therefore, the coefficient associated with price α_i takes two values, $\alpha_i \in \{\alpha^R, \alpha^P\}$. Since the marginal utility of wealth is smaller for rich individuals, we have $0 < \alpha^R < \alpha^P$. Let e_{jk}^R and e_{jk}^P denote cross-price elasticities for the two types. Suppose alternative k happens to have the same market share among the rich individuals and the poor individuals, $\sigma_k^P = \sigma_k^R$. Show that among those who purchase j, the rich individuals are less sensitive to a price cut in k, $e_{jk}^R < e_{jk}^P$.